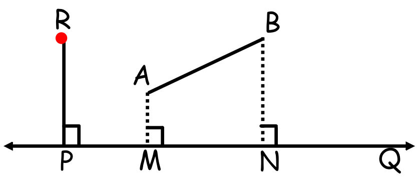
Geometry

Notes 12-5 <u>Proportions in a Right Triangle</u>

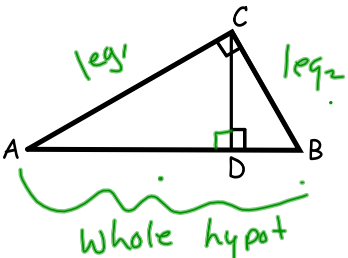
<u>Projection of a point on a line</u>: The foot of the perpendicular drawn from that point to the line.

<u>Projection of a segment on a line</u>: When the segment is not perpendicular to the line, it is the segment whose endpoints are the projections of the endpoints of the given line segment onto the line.

In the figure at the right, $\overline{MN}$ is the projection of the $\overline{AB}$ on line $\overleftrightarrow{PQ}$. The projection of R on line $\overleftrightarrow{PQ}$ is P. If $\overline{PR}$ is perpendicular line $\overleftrightarrow{PQ}$, the projection of $\overline{PR}$ on $\overleftrightarrow{PQ}$ is P.



Theorem 12.16: The altitude to the hypotenuse of a right triangle divides the triangle into two triangles that are similar to each other and to the original triangle.



* The leg is the mean proportional between the whole hypotenuse & the part of the hypotenuse closest to it.

Corollary 12.16a: The length of each leg of a right triangle is the mean proportional between the length of the projection of that leg on the hypotenuse and the length of the hypotenuse.

Referring to the triangle above, $\frac{AB}{AC} = \frac{AC}{AD}$ $\frac{AB}{BC} = \frac{BC}{BD}$

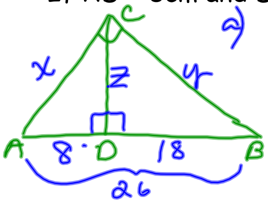
Corollary 12.16b: The length of the altitude to the hypotenuse of a right triangle is the mean proportional between the length of the projections of the legs on the hypotenuse.

"The altitude is the mean proportional between the 2 parts of the hypotenuse."

Referring to the triangle above,

$$\frac{AD}{CD} = \frac{CD}{DB}$$

Ex 1: In right triangle ABC, altitude $\overline{CD}$ is drawn to hypotenuse.
If AD = 8cm and DB = 18 cm, find: **a.** AC **b.** BC **c.** CD



a) $\frac{8}{x} = \frac{x}{26}$

$x^2 = 208$

$x = \pm\sqrt{208}$

$\pm\sqrt{16}\sqrt{13} = 4\sqrt{13}$

$4\sqrt{13}$

b) $\frac{18}{y} = \frac{y}{26}$

$y^2 = 468$

$y = \pm\sqrt{468}$

$\pm\sqrt{36}\sqrt{13}$

$6\sqrt{13}$

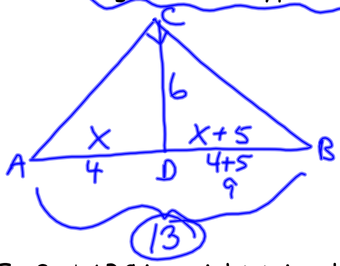
$\frac{8}{z} = \frac{z}{18}$

$z^2 = 144$

$z = \pm 12$

$\boxed{z = 12}$

Ex 2: The altitude to the hypotenuse of right $\triangle$ ABC separates the hypotenuse into two segments. The length of one segment is 5 inches more than the measure of the other. If the length of the altitude is 6 inches, find the length of the hypotenuse.



$\frac{x}{6} = \frac{6}{x+5}$

$x(x+5) = 36$

$x^2 + 5x = 36$

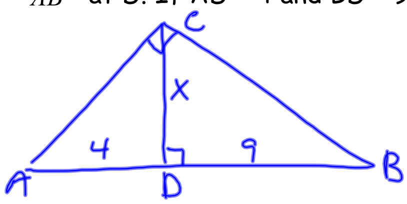
$x^2 + 5x - 36 = 0$

$(x-4)(x+9) = 0$

$x-4=0$ or $x+9=0$

$x = 4$ $\quad$ $x = -9$

Ex 3: $\triangle$ABC is a right triangle with $\measuredangle$ACB the right angle. Altitude $\overline{CD}$ intersects $\overline{AB}$ at D. If AD = 4 and BD = 9, find CD.



$\frac{4}{x} = \frac{x}{9}$

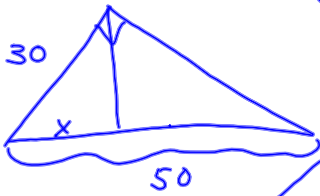
$x^2 = 36$

$x = \pm 6$

$x = 6$

Ex 4: In a right triangle whose hypotenuse measures 50 cm, the shorter leg measures 30 cm. Find the measure of the projection of the shorter leg on the hypotenuse.

Part of hypotenuse closest to shorter leg.

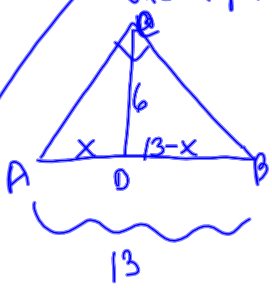


$\frac{x}{30} = \frac{30}{50}$

$\frac{50x}{50} = \frac{900}{50}$

$x = 18$

Rt $\triangle$ ABC with alt. $\overline{CD}$, CD = 6 & AB = 13. Find the 2 parts of $\overline{AB}$



$\frac{x}{6} = \frac{6}{13-x}$

$36 = 13x - x^2$
$\quad -13x + x^2$

$x^2 - 13x + 36 = 0$

$(x-9)(x-4) = 0$